Extreme temperatures in shock-wave interactions with rarefaction waves

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(Received 28 August 1962)

The method of characteristics is used to calculate numerical solutions for the head-on collision of a shock wave with a centred rarefaction wave in the one-dimensional unsteady flow of a perfect gas. It is found that, for sufficiently strong shock waves, the rate of increase in the strength of the shock as it propagates through the rarefaction is so great that the temperature behind the shock reaches extreme values.

1. Introduction

Various aspects of the head-on collision of a shock and a rarefaction wave have been the subject of a number of studies. The general method of Courant & Friedrichs (1948) for the solution of wave-interaction problems in one-dimensional flow may be applied to this problem if the rarefaction is regarded as a discontinuity and, for weak waves, the results of this method have been compared with experiment by Gould (1952). Rosciszewski (1960) applied the characteristics rule of Whitham (1958) to the problem and this method has also been used by Greenspan & Butler (1962), who examined the behaviour of the shock as it nears the gas-vacuum interface of a complete rarefaction wave.

All of these investigations show that the strength of the shock increases as it propagates through the rarefaction and, in the limit, Greenspan & Butler show that the strength becomes infinite at the interface where the gas pressure and temperature drop to zero. However, because of the conflicting effects of the increase in the temperature and pressure ratios across the shock as its strength increases and the fall of the temperature and pressure in front of the shock in the rarefaction, it is not clear whether the temperature and pressure *behind* the shock increase or decrease as it propagates through the rarefaction.

The previous investigations have not resolved this point and the study by the author (Bird 1961) of the related problem of the motion of a shock wave through a region of non-uniform density indicates that the approximate methods cannot be trusted to give reliable results for strong waves. Therefore, the method-of-characteristics computer program which was developed for the previous study has been used to calculate complete numerical solutions of the flow for perfect gases.

2. Description and assessment of methods of analysis

The collision of the shock wave with the rarefaction wave is represented in figure 1 in the distance-time (x, t)-plane. The shock wave of constant shock Mach number† M_{s_0} is moving through a stationary gas of temperature T_0 and pressure p_0 and meets the head of the rarefaction at the point A, which is located at a

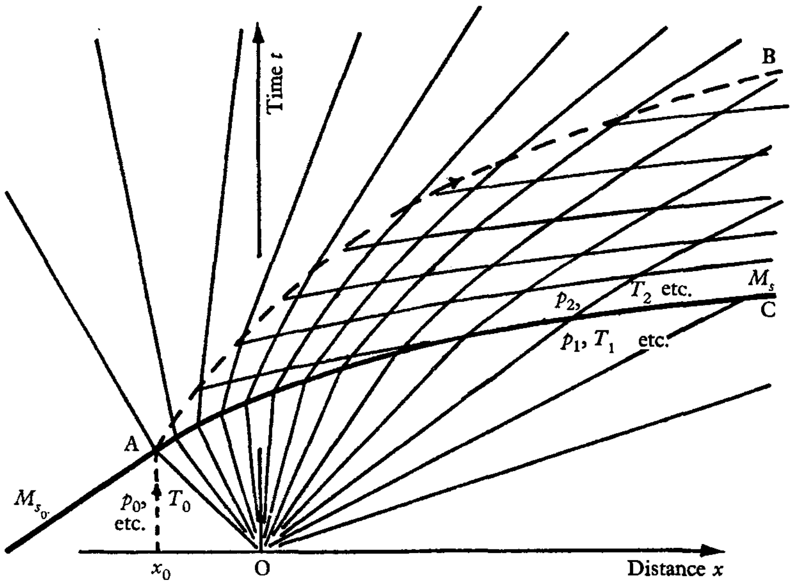

FIGURE 1. Collision of shock wave with centred rarefaction wave.

distance x_0 upstream of the centre O of the rarefaction. The shock Mach number at any point in the rarefaction is denoted by M_s, the flow conditions in the rarefaction immediately in front of the shock are denoted by the subscript 1 and the conditions immediately behind the shock are denoted by the subscript 2.

In order to obtain an exact numerical solution of the flow for a perfect gas, the method of characteristics must be used to construct the flow in the non-isentropic region between the non-uniform portion of shock AC and the particle path AB, the flow behind AB being of the simple wave type. A digital computer program to construct the flow in such a region has been described by the author (Bird 1961) and its adaption to the present problem required only a change in the boundary conditions in front of the shock.

In figure 2, the method-of-characteristics result for a typical case is compared with Rosciszewski's solution which effectively ignores the effects of the positive characteristics in the region ABC. The solution given by the configuration of the wave polars in the pressure-flow velocity plane is also shown. This solution which, as stated earlier, regards the rarefaction as a discontinuity will be called the asymptotic solution because it must give the correct result for the ultimate

† The shock Mach number is defined as the ratio of the speed of the shock relative to the gas in front of it to the speed of sound in this gas.

strength of the waves when the shock meets an incomplete rarefaction followed by constant conditions. The approximate methods give good results for weak rarefaction waves, but diverge widely for more complete rarefactions. The exact characteristics result is closer to Rosciszewski's solution than to the asymptotic solution, although the difference is such that the approximate methods cannot be considered fully satisfactory.

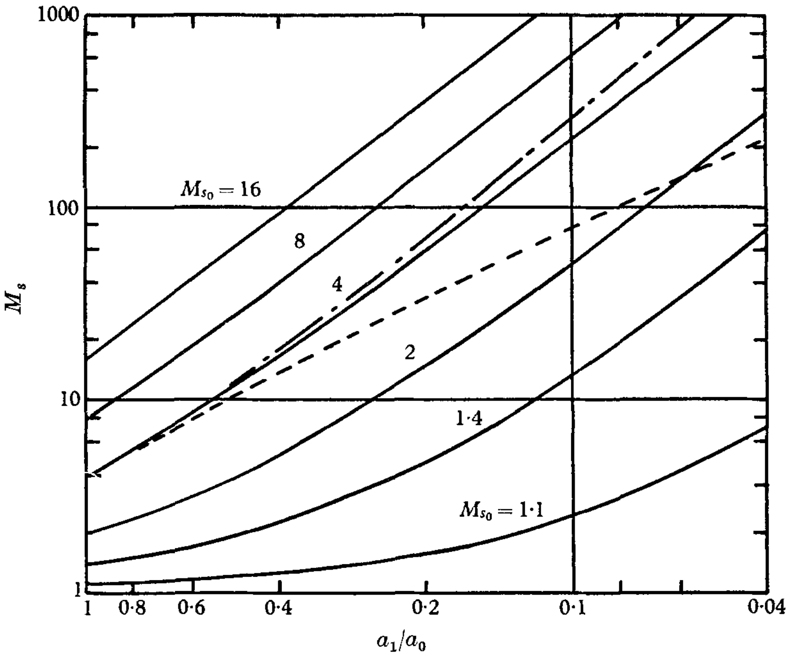

FIGURE 2. Rate of increase of shock strength ($\gamma = \frac{7}{5}$); ——, characteristics; —·—, Rosciszewski; ---, asymptotic.

3. Results and discussion

Figure 2 shows the shock Mach number as a function of the speed of sound in the rarefaction for a number of values of the initial shock Mach number and a perfect gas of $\gamma = \frac{7}{5}$. For very strong waves the curves, which are plotted against logarithmic axes, become almost linear which indicates that, for a given M_{s_0} and a_0,

$$M_s \propto a_1^{-n}.$$

A careful examination of the data indicates that the exponent n is not quite constant over the region of interest. For instance, the results for $M_{s_0} = 4$ indicate that $n = 1·94$ in the region where $M_s = 10^2$ to 10^3 while, for $M_{s_0} = 16$, the corresponding value is 1·96. Also, the initial value of the exponent is 2·03 for $M_{s_0} = 10^2$ and 2·05 for $M_{s_0} = 10^3$. However, for the purposes of this discussion, the value of the exponent will be taken as that given by the initial slope of the curve for $M_{s_0} = 100$ and table 1 presents some results for various values of the specific heat ratio.

For strong shock waves, the speed of sound ratio is proportional to the shock Mach number. Therefore, $a_2/a_1 \propto a_1^{-n}$ and $T_2 \propto a_1^{-2(n-1)}$. Therefore the temperature behind the shock increases as long as $n > 1$ and this is the case for strong shocks in perfect gases with any practical value of γ.

The variation of T_2 with p_1 rather than a_1 is also significant and it is easily shown that

$$T_2 \propto p_1^{-(\gamma-1)(n-1)/\gamma}.$$

Therefore, while the gases with a low value of the specific-heat ratio produce a higher temperature for a given fall in speed of sound in the rarefaction, the specific-heat ratio has little effect on the temperature which is produced for a given pressure drop.

γ	$-n$	$-2(n-1)$	$-\frac{\gamma-1}{\gamma}(n-1)$	$1-\frac{\gamma-1}{\gamma}n$	$-\frac{(\gamma-1)(n-1)}{n-\gamma(n-1)}$
$\frac{5}{3}$	−1·68	−1·36	−0·272	0·327	−0·840
$\frac{7}{5}$	−2·03	−2·06	−0·294	0·420	−0·704
$\frac{11}{9}$	−2·60	−3·20	−0·290	0·494	−0·557
$\frac{19}{17}$	−3·50	−5·00	−0·263	0·631	−0·423

TABLE 1

The pressure ratio across strong shock waves is proportional to M_s^2, so that

$$p_2/p_1 \propto M_s^2 \propto T_2/T_1 \propto T_2^{n/(n-1)}$$

and

$$p_2 \propto p_1^{1-n(\gamma-1)/\gamma}.$$

Table 1 also presents values for this exponent and it is seen that it is always positive so that, in contrast to the temperature, the pressure behind the shock wave always falls.

Finally, the rate of change of temperature behind the shock is given in terms of the pressure variation at this point by

$$T_2 \propto p_2^{-(\gamma-1)(n-1)/\{n-\gamma(n-1)\}}.$$

It is seen from table 1 that, for given M_{s_0}, T_0 and p_0, the gas with the largest specific heat ratio produces the highest temperature at a given pressure.

Figure 2 shows that, for weaker shock waves, the rate of increase of the shock Mach number with the fall in the speed of sound in the rarefaction is less than that for the very strong waves and the behaviour of the temperature behind weaker waves is still in doubt. This point is clarified in figure 3 which shows how the temperature behind the shock changes with distance. Although the temperature behind waves which are initially very weak continues to fall throughout the region of practical interest, there is a dramatic change in the behaviour for M_{s_0} about 2 and, for $M_{s_0} > 4$, the temperature increases almost from the start of the interaction.

The behaviour of the temperature and pressure in front of and behind the shock as a function of the distance from the start of the interaction is shown in figure 4, for an initial shock Mach number of 4 and for several values of the specific-heat ratio. The distance to the high temperature region from the start of the interaction decreases for increasing specific-heat ratio and the monatomic gas exhibits the most favourable over-all behaviour. As the shock becomes stronger, it moves more and more rapidly through the remaining part of the rarefaction and the rate of increase of the temperature behind the shock with distance becomes extremely large. In fact, the curves of figure 4 probably give a good indication of the point at which the shock would overtake the gas-vacuum interface.

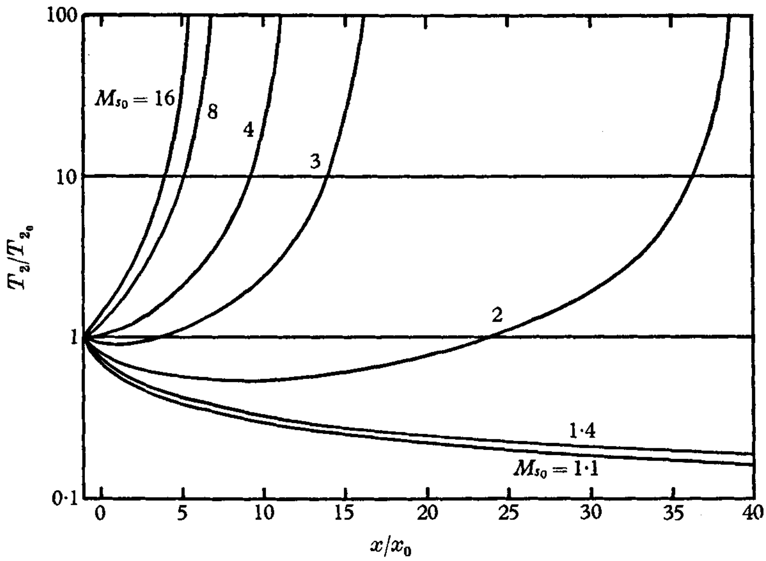

FIGURE 3. Temperature behind shock-wave propagating through rarefaction ($\gamma = \frac{7}{5}$).

In a practical flow the rarefaction would be of finite width and, since the temperature behind the shock at the tail of the rarefaction is much higher than the temperature behind the shock when it reaches its asymptotic strength, it is important to know the rate at which the shock Mach number falls to its asymptotic value. This importance is accentuated by the very short distance in which the major part of the temperature rise takes place. The typical result of figure 5, which is for a shock of $M_s = 4$ propagating through a rarefaction to $a_1/a_0 = 0{\cdot}01$ followed by a uniform flow region, shows that the rate of decrease of the shock Mach number beyond the rarefaction is very slow compared with the rate of increase near the tail of the rarefaction. In a practical flow, the expansion would be to a very small but finite pressure and the tail of the rarefaction would be preceded by a shock wave, as in a normal shock-tube flow, and the pressure at the tail of the rarefaction would be higher than the initial pressure. However, this shock would be quickly overtaken by the main shock and the net result would be similar to that which would be produced if there was a rarefaction right down to the initial pressure.

As an example of the extreme conditions that are predicted by the calculations for perfect gases, consider a shock tube with a monatomic gas at 288 °K and 1 atm in the low-pressure section and a second diaphragm dividing the low-

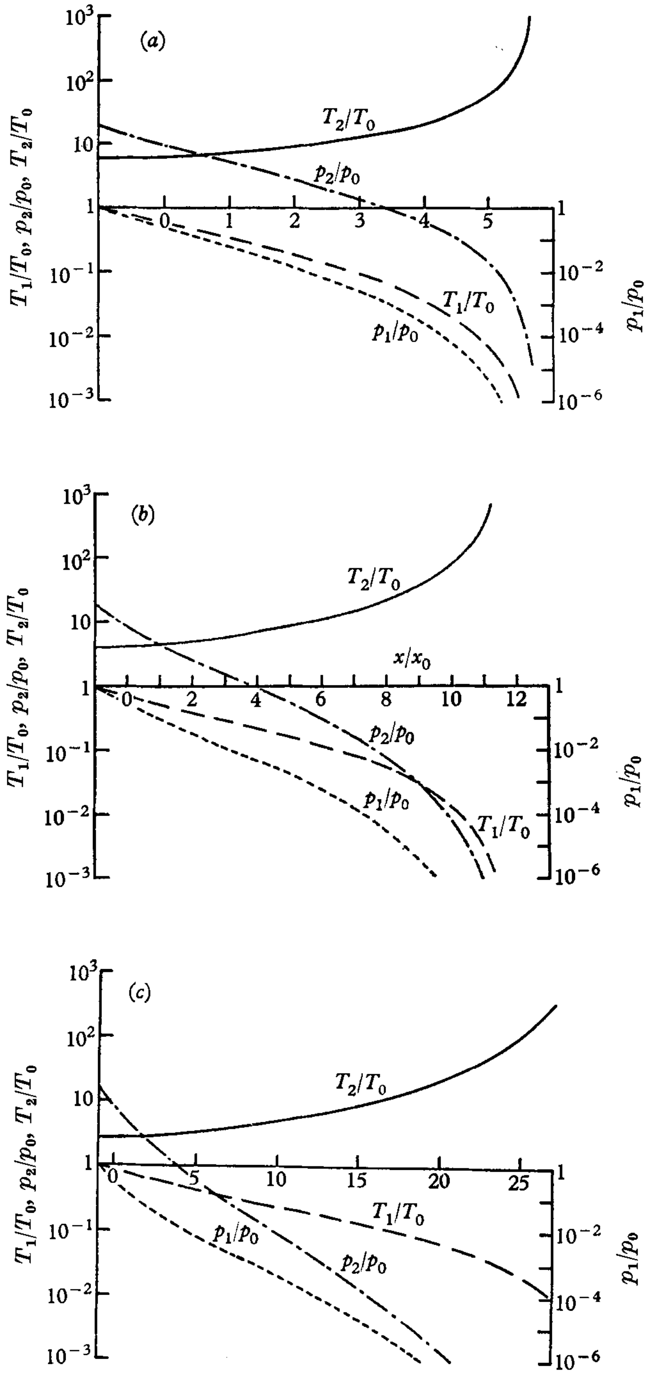

FIGURE 4. Temperature and pressure profiles in typical interactions ($M_{s_0} = 4$). (*a*) $\gamma = \frac{5}{3}$; (*b*) $\gamma = \frac{7}{5}$; (*c*) $\gamma = \frac{11}{9}$.

pressure section from a vacuum. A shock of Mach number 16 is generated in the low-pressure gas and the second diaphragm is broken so that the shock intersects the head of the rarefaction at a distance x_0 upstream of the diaphragm station. Then, at a distance $3{\cdot}436\,x_0$ downstream of this station, a shock of Mach number

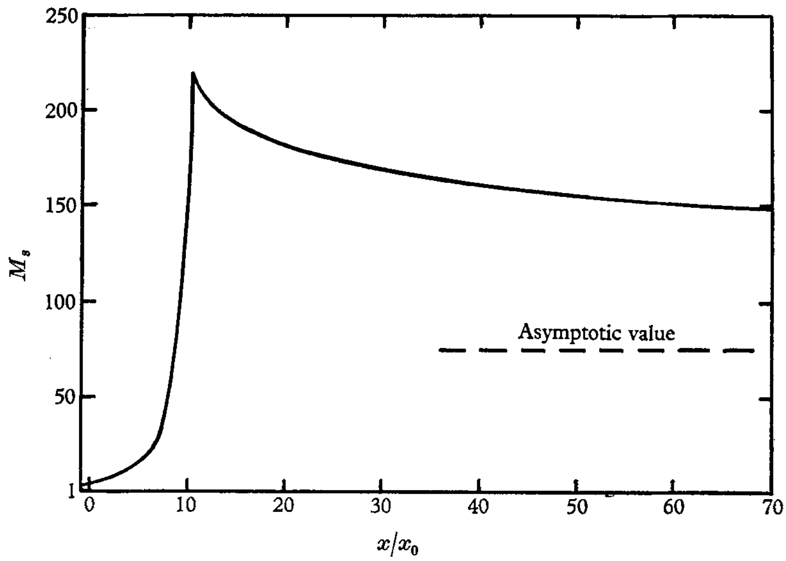

FIGURE 5. Decay in strength of shock wave after passing through an incomplete rarefaction wave.

28,520 would be moving into a gas of temperature $0{\cdot}032\,^\circ$K and pressure $1{\cdot}35\times10^{-10}$ atm which would produce a temperature and pressure behind the shock of $8{\cdot}25\times10^{6}\,^\circ$K and $0{\cdot}137$ atm respectively.

Real gas effects would, of course, drastically modify the results, but the predictions of the perfect gas theory are such that this flow may be expected to provide an effective and comparatively simple method of generating very high temperatures.

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